

An entropy-driven cosmic expansion

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Abstract. We examine the evolution of the Friedman Universe within our recent model of space-time identified with an elastic continuous medium whose deformations are described by a vector field constrained to obey a generalized four-dimensional version of the equilibrium equations of standard elasticity. It is found that the demand that the entropy associated with such elastic deformations be always extremal during the expansion of such a Universe turns these equilibrium equations into a single differential equation governing the evolution of the Hubble parameter H . The solution to the resulting dynamics admits both a power-law expansion, analogous to the one induced by an inflaton field, as well as a power-law expansion analogous to the one induced by a phantom field. Analyzing both types of expansions via the induced elastic energy and pressure permits to assign the former to the early Universe and the latter to its late-time expansion. We discuss the possible way for the dynamics to avoid the Big Rip singularity that would otherwise result. We succinctly discuss the possible way to avoid also the Big Bang singularity and how to obtain the large scale structure of the Universe from the present model.

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1. Introduction

The Hot Big Bang theory, a theory that reproduces marvellously all the actual data from the observed Universe, has led to the well-known initial conditions problem (flatness problem, horizon problem, etc. See e.g. [1]). The idea of the inflationary scenario has rescued the Hot Big Bang theory by providing a mechanism for producing such necessary initial conditions [2, 3, 4] as well as the observed large scale structure of the present Universe [5]. The idea is an early accelerated expansion of the Universe, allowing the latter to achieve the needed initial conditions that make it possible for its subsequent 'normal' expansion to present to us the actually observed picture in the distribution of the CMBR (Cosmic Microwave Background Radiation) in the sky. The idea of an early accelerated expansion of the Universe is usually designated in literature by the word 'scenario' because the details of its mechanism are model dependent since the inflaton field responsible for generating such dynamics is a scalar field entering into some specific Lagrangian models chosen in such a way as to reproduce the precise needed rate and duration of the early expansion.

Another problem that rises within the Hot Big Bang theory when confronted with the observed Universe is the presently observed accelerated expansion of the latter [6, 7, 8]. It is well-known that such expansion could result at the present epoch if there were some kind of dark energy that manifests itself through a cosmological constant leading to a late-time accelerated expansion of the cosmos (see e.g. [9]). But it is also well-known that it is hard to come up naturally with a cosmological constant that is positive and yet as small as the needed one to agree with observations [10]. Many models are also proposed in which one finds another ingredient called a 'phantom' field [11, 12, 13]. Indeed, the possibility that the Universe might actually contain a kind of 'phantom' dark energy is not ruled out by observation (see e.g. [14, 15]). The phantom field may help reproduce the actually observed accelerated expansion but may also lead to a Big Rip singularity in the Universe during a finite time in the future.

Now, it would really be interesting if this idea of an accelerated expansion of the Universe, both in its early times and in its later times, appears as a manifestation of a fundamental principle of nature at work, and intimately linked to the fabrics of space-time itself. The main goal of the present paper is to motivate and examine such a possibility based on our recent work on the idea of an elastic space-time continuum. In fact, many authors have proposed and explored alternative ideas for generating such inflationary and late-time expansions from a modified theory of space-time or some other basic principle. Among many, we find for instance among the recent ones the use of modified Hilbert-Einstein Lagrangian theories (see e.g. [16, 17]), the recent proposal for explaining both inflation and the late-time accelerated expansion in terms of an entropic force [18, 19], and finally attributing the cosmic expansion to an elastic energy stored in space-time and induced by the existence of a cosmic defect in the latter [20, 21]. Our present approach based on the idea of an elastic space-time and its associated entropy emerges from different motivations as to how entropy might lead to a cosmic expansion

(see also [22]) and stands on different constructions concerning how elasticity might enter the dynamics.

Indeed, identifying space-time with an elastic continuous medium and then constructing an entropy functional to be associated with its elastic deformations has proved fertile in the sense that it permits to recover straightforwardly both Einstein field equations and familiar results from black hole thermodynamics [23, 24]. Einstein field equations emerged from the second law of thermodynamics applied to the entropy functional, i.e., they represent a constraint on any elastic space-time whose elastic deformations always extremize the entropy associated with them. Black hole thermodynamics formulas, i.e., the Hawking temperature and the Bekenstein-Hawking entropy formula, are derived from the same entropy functional when the deformation vector field u^i contained in the latter is constrained to obey a generalized four-dimensional version of the equilibrium equations of standard elasticity [24]. That is, the second principle of thermodynamics is responsible for the emergence of the dynamics as well as the thermodynamics of such space-times †. We shall see in the present paper that the same principle may also be behind the dynamics of the observable Universe as a whole and may lead to the crucial early accelerated expansion of the Universe without appealing to an inflaton field and produce a late-time acceleration without appealing to a cosmological constant or a phantom field.

The outline of this paper is as follows. In section 2, we recall the main ingredients that allowed us to arrive at Einstein field equations by identifying space-time with an elastic continuum medium and then use them to analyze the dynamics of the Friedman Universe corresponding to such a space-time. In section 3, we compute explicitly and examine in detail the two solutions to the dynamical equation obtained for the Hubble parameter H in section 2. In section 4, we examine the elastic energy and pressure induced by the elastic deformations and propose a scenario for how and why the Universe might undergo an inflationary expansion early in its history before reaching a radiation and then matter dominated eras. We then discuss how it might have entered its late-time evolution that looks like a phantom dark energy-dominated era. We end this paper with a general conclusion discussing the possible way to avoid the Big Bang singularity and the way the large scale structure of the Universe could emerge from the model.

2. Entropy and the dynamics of a Friedman elastic Universe

In our elastic space-time approach [24], following ref. [23], we considered space-time to be a continuum medium whose elastic deformations are quantified by the vector field $u^i = \bar{x}^i - x^i$, where $i = 0, \dots, 3$ and \bar{x}^i and x^i are coordinate labels in the space-time after and before deformation, respectively. An entropy, which is a functional of this deformation vector field, is then constructed to be associated with these deformations.

† The approach, when generalized to Riemann-Cartan space-times, also permits to recover the Cartan-Sciama-Kibble field equations [25].

The most general covariant form obtained for the functional was [23, 24]

$$\mathcal{S} = \frac{1}{8\pi G} \int_{\mathcal{M}} d^4x \sqrt{-g} \left[\nabla_i u_j \nabla^j u^i - (\nabla_i u^i)^2 + 8\pi G \left(\frac{1}{2} g_{ij} T - T_{ij} \right) u^i u^j \right]. \quad (1)$$

Here, the eventual cosmological constant that the functional (1) might contain is omitted in order to investigate the dynamics of the Universe without a cosmological constant. This functional was then varied with respect to the field u^i by imposing $\delta\mathcal{S} = 0$ for all possible deformations u^i . The resulting equations are nothing but the Einstein field equations $R_{ij} - \frac{1}{2} g_{ij} R = -8\pi G T_{ij}$. Substituting these back into the original functional (1) gives an 'on-shell' entropy functional that reads, after integration by parts [23],

$$\mathcal{S} = \frac{1}{8\pi G} \int_{\partial\mathcal{M}} d^3x \sqrt{|h|} n_i (u^j \nabla_j u^i - u^i \nabla_j u^j), \quad (2)$$

where h is the determinant of the three-metric on the hypersurface bounding the integrated region of space-time. At this point, nothing is imposed yet on the vector field u^i ; Einstein equations emerge only from imposing $\delta\mathcal{S} = 0$ for whatever values the field u^i might happen to take. That is, the construction suggests that the dynamics of the metric of space-time (i.e., gravity) emerges from a deeper level in the structure of space-time at which the latter always tends to satisfy the second law of thermodynamics. However, taking this vector field to be a kind of 'vector-state' representing the difference between the two states of space-time after and before deformation as in elasticity theory, naturally suggests to picture each infinitesimal volume element of space-time as if it were in equilibrium within the whole continuum due to the elastic stresses it experiences from its surrounding neighbours. The simplest (and heuristic) way to express the equilibrium condition of these elements of space-time is then to adopt a four-dimensional generalization of the Hooke's law $\sigma_{\alpha\beta} = \mu \delta_{\alpha\beta} \varepsilon_{\gamma}^{\gamma} + 2\nu \varepsilon_{\alpha\beta}$, giving the relation between the stress tensor $\sigma_{\alpha\beta} = \sigma_{\beta\alpha}$ and the strain tensor $\varepsilon_{\alpha\beta} = \frac{1}{2}(\partial_{\alpha} u_{\beta} + \partial_{\beta} u_{\alpha})$, as well as a four-dimensional generalization of the usual equilibrium equations of elasticity $\partial_{\alpha} \sigma^{\alpha\beta} = 0$ expressed in terms of the stress tensor in the absence of external and non-elastic forces ($\alpha, \beta = 1, 2, 3$) (see e.g. [26]). In standard elasticity, the above form of the Hooke's law applies whenever the continuum in consideration is homogeneous and isotropic as we shall suppose it to be the case for space-time §. The positive coefficients μ and ν are called the Lamé coefficients and describe the elastic properties of the continuum. The stress tensor $\sigma^{\alpha\beta}$ represents the force (per unit area) applied on an infinitesimal volume element of the continuum, pointing in the direction α and perpendicular to a surface whose normal is in the direction β . In addition, when space-time is not considered as embedded in a higher-dimensional manifold, one may also want to generalize the vanishing of the rigid-rotations tensor $\omega_{\alpha\beta} = \frac{1}{2}(\partial_{\alpha} u_{\beta} - \partial_{\beta} u_{\alpha})$ in ordinary elasticity to four dimensions. The equations one obtains when all these ingredients are transcribed into a covariant four-dimensional form are [24]

$$\sigma^{ij} = \mu g^{ij} \nabla_k u^k + 2\nu \nabla^i u^j, \quad (3)$$

§ For an elaborate discussion on the tensors of anisotropic elasticity see [27].

for the generalized Hooke's law, and

$$\mu(\nabla^i \nabla_j u^j) + 2\nu(\nabla_j \nabla^i u^j) = 0, \quad (4)$$

for the generalized equilibrium equations. Here μ and ν would be the positive Lamé elastic coefficients of space-time. Using the Ricci identity $[\nabla_j, \nabla_i]u^j = R_{ij}u^j$ in curved space-times, where the symmetric Ricci tensor R_{ij} is constructed from the metric g_{ij} and the Riemann tensor $R_{ij} = g^{kl}R_{kilj}$, the above equations also read

$$\nabla_i \nabla_j u^j = -\frac{2\nu}{\mu + 2\nu} R_{ij} u^j. \quad (5)$$

In standard elasticity, the Lamé coefficients depend on temperature and the density of defects inside the elastic material. In our approach, matter and pure energy are viewed as dislocations (or defects) in the space-time continuum. As such, we may also expect non-trivial modifications of these equilibrium equations when the density of defects (i.e., matter or radiation) becomes non-negligible. We shall discuss below in detail the consequences of this assumption within the framework of the present model.

All that remains now is to specify the space-time we wish to study and apply these two constraints, namely, that its entropy due to its elastic deformations must be given by (2) in order for it to have a constantly extremized value and its deformations obey (5) in order for it to be a continuum in equilibrium at everyone of its points. The specific space-time we shall consider hereafter is the homogenous and isotropic Friedman Universe described by the Friedman-Lemaître-Robertson-Walker (FLRW) metric

$$dl^2 = -dt^2 + a^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2 \right). \quad (6)$$

$a(t)$ is the time-dependent scale factor and $k = -1, 0, +1$ is the parameter that distinguishes the open, flat, and closed Universes, respectively. Now in order to deduce the dynamics that governs the time evolution of such a space-time, i.e., the scale factor $a(t)$, one usually injects into the left-hand side of Einstein equations the Einstein tensor $R_{ij} - \frac{1}{2}g_{ij}R$ that comes out from the selected space-time (6) while in the right-hand side one injects the energy-momentum tensor $T_{ij} = \text{diag}(\rho, -p, -p, -p)$ that comes from the radiation or the matter filling that space-time. In the resulting dynamics, described by the Friedman-Lemaître equations $\dot{a}^2/a^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$ and $\ddot{a}/a = -\frac{4\pi G}{3}(\rho + 3p)$, the density ρ and the pressure p of matter or radiation are the unique sources capable of inducing an expansion for the Universe. The radiation-dominated era (for which the equation of state is $p/\rho = w = \frac{1}{3}$) gives $a(t) \sim t^{1/2}$ whereas the matter (in the form of dust)-dominated era (for which $p/\rho = w = 0$) yields $a(t) \sim t^{2/3}$. The two equations may actually be combined after computing the first derivative of each and then using the definition $\dot{a}/a = H$ of the Hubble parameter as well as the identity $\ddot{a}/a = \dot{H} + H^2$. The result is the following single differential equation to which we shall refer below,

$$-\frac{\ddot{H}}{H^3} - (5 + 3w)\frac{\dot{H}}{H^2} = 3(1 + w). \quad (7)$$

Thus, for all three possible values of k , the Hubble parameter H cannot induce a power-law expansion for $a(t)$ of the form A/t in early-times or $B/(t_0 - t)$ (with $t_0 > t$) in

later-times unless the equation of state of the sources entering (7) is $-1 < w < 0$ or $w < -1$, respectively. The first solution gives rise to what is known as a power-law inflation, whereas the second induces the phantom behaviour. This is where the need for an inflaton and then a phantom field comes from. Given that our approach consisting of assigning to space-time an entropy related to its elastic deformations and which, when extremized, yields its dynamics it is tempting to think that this same approach may help follow the dynamics of space-time through its whole evolution beginning from its early times immediately after the Planck era and all the way through to its late-time expansion without appealing to inflaton fields for early times and phantom fields for later times. In other words, instead of investigating the evolution of the Universe using Einstein equations (i.e., the 'metric' gravity) that tell us how matter and energy when they dominate influence the dynamics of the metric through the Friedman-Lemaître equations we, in a sense, descend into a deeper level and check directly the deformations of space-time (the 'entropic' elastic gravity) that gave birth in the first place, through the second law of thermodynamics, to the dynamics of the metric. At this level it is legitimate, in principle, to expect that there would be no need for particular treatments for each of the four different epochs in the history of the Universe. We shall come back to this last point later.

We begin then from the usual assumptions of homogeneity and isotropy of space. First, from the assumption of homogeneity of space we expect only time-dependent components of the field u^i whereas isotropy suggests to have only a time-component u^0 of the field u^i and possibly also a radial component u^1 . That is, we shall start with the following ansatz for the deformation vector field

$$u^i = (u^0, u^1, 0, 0) \equiv (\phi(t), \varrho(t), 0, 0). \quad (8)$$

As a consequence, the resulting 'on-shell' entropy takes the following form

$$\begin{aligned} \mathcal{S} = & -\frac{1}{8\pi G} \int \frac{a^3 r^2 \sin \theta}{\sqrt{1 - kr^2}} (u^1 \nabla_1 u^0 - u^0 \nabla_1 u^1) dr d\theta d\varphi \\ & + \frac{1}{8\pi G} \int \frac{a^3 r^2 \sin \theta}{\sqrt{1 - kr^2}} (u^0 \nabla_0 u^1 - u^1 \nabla_0 u^0) dt d\theta d\varphi. \end{aligned} \quad (9)$$

Next, let us start from the FLRW metric (6) and obtain the precise equilibrium equations that result for the field u^i . Although there are in (5) four equilibrium equations in all, due to the form (8) of the deformation field only two equations remain. They are

$$\partial_t(\dot{\phi} + \Gamma_{j0}^j \phi + \Gamma_{j1}^j \varrho) = -\frac{2\nu}{\mu + 2\nu} (R_{00}\phi + R_{01}\varrho), \quad (10)$$

$$\partial_r(\dot{\phi} + \Gamma_{j0}^j \phi + \Gamma_{j1}^j \varrho) = -\frac{2\nu}{\mu + 2\nu} (R_{10}\phi + R_{11}\varrho). \quad (11)$$

The needed non-vanishing Christoffel symbols $\Gamma_{\alpha 0}^\alpha$ and $\Gamma_{\alpha 1}^\alpha$ ($\alpha = 1, 2, 3$) as well as the non-vanishing components R_{00} and R_{11} of the Ricci tensor are extracted from the FLRW metric (6). They are, respectively,

$$\Gamma_{10}^1 = \Gamma_{20}^2 = \Gamma_{30}^3 = \frac{\dot{a}}{a}, \quad \Gamma_{21}^2 = \Gamma_{31}^3 = \frac{1}{r}, \quad (12)$$

$$R_{00} = \frac{3\ddot{a}}{a}, \quad R_{11} = -\frac{a\ddot{a} + 2\dot{a}^2 + 2k}{1 - kr^2}. \quad (13)$$

Substituting these in (10) and (11), the equilibrium equations take the following more explicit expressions

$$\partial_t(\dot{\phi} + 3\frac{\dot{a}}{a}\phi + \frac{2}{r}\varrho) = -\frac{6\nu}{\mu + 2\nu}\frac{\ddot{a}}{a}\phi, \quad (14)$$

$$\partial_r(\dot{\phi} + 3\frac{\dot{a}}{a}\phi + \frac{2}{r}\varrho) = \frac{2\nu}{\mu + 2\nu}\frac{a\ddot{a} + 2\dot{a}^2 + 2k}{1 - kr^2}\varrho. \quad (15)$$

Now, performing the r -derivative in the left-hand side of the last equation and then transposing the denominator $1 - kr^2$ in the right-hand side to the left, we immediately see that the only way for the two sides – one having a multiplicative factor that depends only on the variable r whereas that of the other depends only on the variable t – to be equal is to have an identically vanishing $\varrho(t)$. So we conclude that the field u^i that is compatible with the FLRW geometry and simultaneously constrained to obey the generalized equilibrium equations (4) should be of the form $u^i = (\phi(t), 0, 0, 0)$. The only remaining constraint to be imposed on this field is therefore Eq. (14). Performing the time-derivative and using $H = \dot{a}/a$ and $\ddot{a}/a = \dot{H} + H^2$ in order to recast the equation in terms of the Hubble parameter H instead of the scale factor a , the constraint equation becomes the following dynamical equation for ϕ

$$\ddot{\phi} + 3H\dot{\phi} + \left(\frac{3\mu + 12\nu}{\mu + 2\nu}\dot{H} + \frac{6\nu}{\mu + 2\nu}H^2\right)\phi = 0. \quad (16)$$

We notice a form similar to the familiar dynamical equations usually obtained for the scalar ϕ in the scalar field models approach to inflation and phantom dark energy, but with a potential energy that is a H and \dot{H} -dependent function. Going back to (9), the 'on-shell' functional associated with the field $(\phi(t), 0, 0, 0)$ simplifies further to

$$\mathcal{S} = \frac{3\dot{a}a^2\phi^2}{8\pi G} \int \frac{r^2 \sin \theta}{\sqrt{1 - kr^2}} dr d\theta d\phi = \frac{3\mathcal{V}_r}{8\pi G} \dot{a}a^2\phi^2, \quad (17)$$

where \mathcal{V}_r is the constant $\int \frac{r^2 \sin \theta}{\sqrt{1 - kr^2}} dr d\theta d\phi$ representing the co-moving coordinate-volume. We see from this functional that in order for the entropy associated with the elastic deformations of space-time to be strictly positive and always extremal, in accordance with the second law of thermodynamics, the scale factor should be non-vanishing and increasing with time to insure $\dot{a} > 0$. Furthermore, the scalar field ϕ should absorb any such increase of the scale factor a as well as its derivative \dot{a} in (17). That is, the scalar ϕ should take the following form

$$\phi \sim \frac{1}{a\sqrt{\dot{a}}} = \frac{1}{a^{3/2}\sqrt{H}}, \quad (18)$$

the Hubble parameter H being strictly positive as it follows from the argument above. Substituting this form of the field ϕ in Eq. (16), the latter becomes the following nonlinear differential equation for the Hubble parameter H

$$-\frac{\ddot{H}}{H^3} + \frac{3}{2}\frac{\dot{H}^2}{H^4} + 3\lambda\frac{\dot{H}}{H^2} = \frac{3}{2}\eta, \quad (19)$$

where $\lambda = (\mu + 6\nu)/(\mu + 2\nu)$ and $\eta = (3\mu - 2\nu)/(\mu + 2\nu)$. This is the equation that governs, in our model, the dynamics of the elastic Friedman Universe. It looks like Eq. (7) obtained from the Friedman-Lemaître equations, except for the one additional term \dot{H}^2/H^4 . However, in contrast to Eq. (7) this last equation allows solutions for the Hubble parameter H of the form A/t as well as $B/(t_0 - t)$ without introducing matter sources whose equations of state should satisfy $w < 0$. The trick is done by the additional term \dot{H}^2/H^4 as well as the the Lamé coefficients whose role is to influence the behavior of the elastic deformations. In the next section, we examine the detailed form of these two solutions of the equation, relate them respectively to the inflationary expansion and the late-time expansion of the Universe, and then discuss the intermediate radiation and matter-dominated eras.

3. From inflation to late-time acceleration

3.1. Inflationary expansion

When $\eta \neq 0$, Eq. (19) admits the following two solutions $H = A/(t_0 + t)$ or $H = B/(t_0 - t)$ for some arbitrary constants A , B , and t_0 . The case $\eta = 0$ will be discussed in the last section of this paper. We begin here by examining the first one after shifting the time $t_0 + t \rightarrow t$ for simplicity. Inserting $H = A/t$ into (19) we find two possible values for A :

$$A = \frac{-3\lambda \pm \sqrt{9\lambda^2 - 3\eta}}{3\eta}. \quad (20)$$

Remembering, though, that $\eta = (3\mu - 2\nu)/(\mu + 2\nu)$, we have also to distinguish between $\eta < 0$ when $\mu < 2\nu/3$ and $\eta > 0$ when $\mu > 2\nu/3$. For positive η , however, the two solutions (20) both yield negative values for the constant A and hence also negative values for the Hubble parameter H . Thus, only $\mu < 2\nu/3$ permits to have $H > 0$ and one must choose the solution with the minus sign in (20). Furthermore, when $|\eta| \ll 1 < \lambda$ the following approximation $A \approx -2\lambda/\eta$ holds, and the corresponding time-dependence of the scale factor is

$$a(t) \propto t^{-\frac{2\lambda}{\eta}} = t^{\frac{2(\mu+6\nu)}{2\nu-3\mu}}. \quad (21)$$

We see that we can get a sufficient inflationary expansion within this model, and more precisely the required minimum amount of 70 e-foldings [5], provided that the Lamé coefficients μ and ν satisfy $32\nu/53 \leq \mu < 2\nu/3$. But it is easy to see that for $0 < \mu < 2\nu/3$ we may recover all the preferred exponents $-\frac{2\lambda}{\eta} \gg 10$ of power-law inflation [28]. Next, we shall investigate the implication of this approximation on the other solution, namely when $H = B/(t_0 - t)$.

3.2. Late-time expansion

Inserting the other solution $H = B/(t_0 - t)$ into Eq. (19) we also find two possible values for B :

$$B = \frac{3\lambda \pm \sqrt{9\lambda^2 - 3\eta}}{3\eta}. \quad (22)$$

This time, however, both $\eta > 0$ and $\eta < 0$ permit to have positive values for B and hence a positive H . Working with the fixed choice $\mu < 2\nu/3$ that gives a negative η , and hence a positive $H = A/t$ for the first solution, we must choose the minus sign in (22) in order to have again a positive H for this case. We verify then that the approximation $|\eta| \ll 1 < \lambda$ that allows us to find an inflationary expansion gives the moderate coefficient $B \approx 1/(6\lambda)$, which in turn induces the following time-dependence for the scale factor

$$a(t) \propto \frac{1}{(t_0 - t)^{\frac{1}{6\lambda}}} = \frac{1}{(t_0 - t)^{\frac{\mu+2\nu}{6(\mu+6\nu)}}}. \quad (23)$$

For the case $32\nu/53 \leq \mu < 2\nu/3$ this becomes, in the lower limit, approximately $a \propto (t_0 - t)^{-0.07}$. The exponent is, however, not sufficient compared to what actual observations suggest for the phantom power-law [29], but this is the best we can achieve in this model with fixed Lamé coefficients. Expression (23) obviously implies the Big Rip singularity in the finite time t_0 for any positive value of λ . We shall come back to this fact below. But for now it is satisfactory to have found both the inflaton field-induced behavior as well as the phantom dark energy one within the same dynamical equation and within the same approximation scheme for both.

3.3. Radiation and matter-dominated eras

As for getting the intermediate radiation and matter-dominated eras, one might also be tempted to seek a solution for the corresponding dynamics using the same equation (19). However, identifying the latter with Eq. (7) which is valid for these eras leads one to impose negative values on the Lamé coefficients and, even more, different ones for each of the two eras. The reason for the failure of Eq. (19) to take into account the presence of matter and radiation is the fact that, as we have alluded to it in section 2, matter and radiation are viewed as defects in the space-time continuum that spoil the usual equilibrium equations of elasticity in their neighborhoods. To find the deformation field in the presence of these requires taking into account the presence of defects when writing down the equilibrium equations themselves and this should be done, not within a generalization of the linear three-dimensional elasticity theory, but within the framework of a generalized version of the theory of defects in crystals [30]. That can also, in principle, be done by building a model in which the density of defects of space-time intervenes in the equilibrium equations. That is however beyond the scope of our present model, and so the latter still requires the Friedman-Lemaître equations (i.e., the 'metric' gravity) to treat radiation and matter-dominated eras properly.

4. Distinguishing inflation from late-time expansion

At this point, and using Eq. (19) alone, there is yet no way to distinguish between the dynamics of the early-times and the dynamics of the late-times. So there is no reason to assign the solution $H = A/t$ to the former and $H = B/(t_0 - t)$ to the latter. Indeed,

one may very well take instead the first solution to represent a late-time quintessential power-law expansion [17, 29], but then one is left with no other choice for early-time inflation. Examining the elastic energy of space-time as well as the elastic pressure, both induced by the elastic deformations, provides complementary information on the dynamics, however.

When generalizing the symmetric stress tensor $\sigma^{\alpha\beta}$ ($\alpha, \beta = 1, 2, 3$) of standard elasticity to four dimensions, the resulting symmetric tensor σ^{ij} acquires the new components $\sigma^{\alpha 0}$ and σ^{00} that we don't find in three dimensions. Since $\sigma^{\alpha\beta}$ represents in standard elasticity a force per unit area, applied in the direction α and perpendicular to the surface element whose normal is in the direction β , we shall interpret the components of the generalized stress tensor as follows. The component $\sigma^{\alpha\beta}$ would represent the force per unit area, applied in the direction α and perpendicular to the elementary hypersurface $dx^2 dt$ whose normal is in the direction β . That is, $\sigma^{\alpha\beta}$ is the pressure per unit cosmic time on a two dimensional surface. The component $\sigma^{\alpha 0}$ would represent the energy contained in an elementary hypersurface $dx^2 dt$ whose normal is in the direction α . That is, $\sigma^{\alpha 0}$ is a flux of energy through a two dimensional surface. The component σ^{00} then would be the energy stored inside the elementary hypersurface dx^3 whose normal is in the time direction, i.e., the three-dimensional spatial volume dV . That is, σ^{00} is the elastic energy density. Using Eq. (3), the non-vanishing Christoffel symbols (12), and the expression (18) for the field ϕ , we arrive at the following identities that are also expressed in terms of the scale factor a and the Hubble parameter H

$$\sigma^{\alpha 0} = 0, \quad (24)$$

$$p \equiv \frac{1}{3}\sigma_{\alpha}^{\alpha} = \mu\dot{\phi} + (3\mu + 2\nu)H\phi = \frac{\sqrt{H}}{2a^{3/2}}\left[\mu\left(3 - \frac{\dot{H}}{H^2}\right) + 4\nu\right], \quad (25)$$

$$\mathcal{E} \equiv \sigma^{00} = -(\mu + 2\nu)\dot{\phi} - 3\mu H\phi = \frac{-\sqrt{H}}{2a^{3/2}}\left[\mu\left(3 - \frac{\dot{H}}{H^2}\right) - 2\nu\left(3 + \frac{\dot{H}}{H^2}\right)\right]. \quad (26)$$

We first notice the vanishing of the flux of energy as perceived in a co-moving reference frame through any two dimensional spatial surface, as one would expect from our assumption that space-time is not imbedded in a higher dimensional manifold. Indeed, one might expect possible energy flows only if there were extra-dimensions outside the four-dimensional space-time; an eventuality that we won't consider here. The energy density and pressure are, however, not null and we will compute their corresponding values separately for each of the two solutions found for H from Eq. (19).

For $H = A/t$ we have $\dot{H} = -H^2/A$ with A given by (20) which, in the approximation $|\eta| \ll \lambda$, reduces to $-2\lambda/\eta$, so that $\dot{H} \approx \eta H^2/2\lambda$. Substituting this in the above expressions for pressure and energy density, these read

$$p \approx \frac{\sqrt{H}}{2a^{3/2}}(3\mu + 4\nu), \quad (27)$$

$$\mathcal{E} \approx \frac{-\sqrt{H}}{2a^{3/2}}(3\mu - 6\nu). \quad (28)$$

Recalling that H comes out positive when $\mu < 2\nu/3$, we notice that both expressions above are positive. Hence, for the solution $H = A/t$ the pressure pushes outwards and the energy density is positive.

For $H = B/(t_0 - t)$ we have $\dot{H} = H^2/B$ with B given by (22) which, in the approximation $|\eta| \ll \lambda$, reduces to $1/(6\lambda)$, so that $\dot{H} \approx 6\lambda H^2$. Substituting this in the above expressions for pressure and energy density, these read

$$p = -\frac{\sqrt{H}}{2a^{3/2}} \frac{3\mu^2 + 26\mu\nu - 8\nu^2}{\mu + 2\nu}, \quad (29)$$

$$\mathcal{E} = \frac{\sqrt{H}}{2a^{3/2}} \frac{3\mu^2 + 48\mu\nu + 84\nu^2}{\mu + 2\nu}. \quad (30)$$

We notice that the energy density is positive whereas the pressure comes out negative in the range $32\nu/53 \leq \mu < 2\nu/3$. Hence, for the solution $H = B/(t_0 - t)$ the pressure pulls inwards even if the energy density is positive and the Universe expands. Before drawing the corresponding picture for the two accelerated phases of the cosmic expansion that emerges from this analysis, we also compute the total elastic energy $E = \int \mathcal{E} dV = a^3 \mathcal{V}_r \mathcal{E}$ from the above energy densities corresponding to each solution. Omitting the constant factors, we write only the time-dependence of these and display beside each one of them the corresponding time-dependence of the energy-density and the field $\phi \sim a^{-3/2} H^{-1/2}$. For $H = A/t \approx -2\lambda/(\eta t)$, we get

$$E \sim t^{\frac{3\lambda}{|\eta|}}, \quad \mathcal{E} \sim t^{-\frac{3\lambda}{|\eta|}}, \quad \phi \sim t^{-\frac{3\lambda}{|\eta|}}. \quad (31)$$

While for $H = B/(t_0 - t) \approx 1/[6\lambda(t_0 - t)]$, we find

$$E \sim (t_0 - t)^{-\frac{1+2\lambda}{4\lambda}}, \quad \mathcal{E} \sim (t_0 - t)^{\frac{1+2\lambda}{4\lambda}}, \quad \phi \sim (t_0 - t)^{\frac{1+2\lambda}{4\lambda}}. \quad (32)$$

During both accelerated expansions $H = A/t$ and $H = B/(t_0 - t)$ energy increases whereas the field ϕ decreases even though the Universe expands. However, during the phase $H = A/t$ energy density decreases while during the phase $H = B/(t_0 - t)$ energy density increases. It is this crucial difference between the behavior of the elastic energy density during the two types of expansion that induces us to assign the former to the early-times while reserving the latter to late-times. Indeed, the possible interpretation of this fact is the following. When elastic energy increases at the same time that energy density decreases induces the subsequent formation of radiation and matter in the form of defects and dislocations in the continuum of space-time, i.e., the excess of elastic energy is transformed into radiation and matter whose formation is favored both by the energy density decrease and the decrease of ϕ . This would be the analogue in this model of the reheating process. The radiation and matter are thus nothing but an elastic energy stored in the form of dislocations and defects. When the density of these becomes non-negligible, the subsequent evolution of the Universe is more properly described as we saw above by the 'metric' gravity through the Friedman-Lemaître equations. When the density of matter then falls off considerably due to cosmic expansion, the 'elastic' gravity turns on again, but this time starts out with the value of ϕ acquired just before entering the phase $H = B/(t_0 - t)$. Since the subsequent decrease of ϕ is of the form

$(t_0 - t)^{(1+2\lambda)/4\lambda}$, putting $t = 0$ and equating that with the value of ϕ acquired at the end of the matter-dominated era – that would be given by a model including dislocations properly – would yield the value of t_0 .

Now, the latter form of expansion is usually ascribed to a phantom dark energy, for which the Universe might actually go towards the Big Rip singularity in the finite time t_0 . The present model, however, suggests a way out. Indeed, during such expansion we have a negative pressure that pulls inwards, a deformation ϕ that decreases faster than the increase of the scale factor, and also an energy density that increases faster than the scale factor. Thus, when reaching the Planck scales quantum mechanical effects would begin to dominate before the scale factor has any chance to blow up. It is then not excluded that a quantum mechanical behaviour of the field ϕ may prevent the Universe from following its late-time dynamical regime until it reaches the singularity.

5. Conclusions

In this paper we studied the dynamics of the Universe in the framework of the assumption that space-time is an elastic continuum whose deformations obey generalized equilibrium equations of three-dimensional elasticity and whose entropy is always extremal. We have seen that it is possible to recover both the inflationary expansion of the Universe and a late-time phantom-like behavior without appealing to inflaton and phantom fields. There is within this model still an alternative possibility, however, which is to have a de Sitter Universe and a cosmological constant. Indeed, when $\eta = 0$ the dynamical equation (19) admits as a solution only a constant Hubble parameter H . But this would yield the same exponential expansion $a \sim \exp(Ht)$ for the Universe both for inflation and in later-times, unless one finds a mechanism that changes the constant Hubble parameter when going through radiation and then matter-dominated eras within this model. This is why the possibility for η to be null is not much dwelled upon here.

Another great merit of the scalar field-based inflationary scenario, however, is its possibility to provide also the observed large scale structure of the Universe by studying the primordial quantum fluctuations of the inflaton field that get magnified to the presently observed ones through the rapid expansion of the very early Universe [5]. It is, in principle, also possible to conduct a similar analysis within the present model by considering the quantum fluctuations of the field ϕ . But since the latter is still viewed as a classical field in our approach, the study of its eventual quantum mechanical origins and quantum fluctuations are beyond the scope of the present work and will be deferred to future investigations.

Finally, we see that a vanishing or a decreasing scale factor $a(t)$ are both forbidden in the framework of our present model since Eq. (17) would then make entropy either vanish or come out negative. This constitutes a hint that the original Big Bang singularity may also be avoided within this approach. But since that issue pertains to the realm of the Planck era, a proper treatment of that question may again only come from a quantum

mechanical model for the field ϕ .

6. References

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